# UNDERSTANDING THE CONCEPT OF ALGEBRAIC ANNIHILATING IDEAL GRAPH OF A COMMUTATIVE RING

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***Abstract***

*The rings considered in this article are commutative with identity which concedes at any rate one nonzero annihilating ideal. The structure of commutative rings with the assistance of annihilating ideal graphs and some related graphs we have related a graph to a given ring R and considered the interplay between the structure of R related with it. We have considered a few points related with a commutative ring whose vertices are nonzero annihilating ideals of the ring. So in this paper we will talk about the algebraic ideal graph of a commutative ring with the connectivity and finiteness conditions.*

***Keywords: Graph, Algebra, Ring theory, ideal, commutative ring, etc.***

## 1. INTRODUCTION

Algebra is the principal part of mathematics. It includes the investigation of algebraic structure, for example, gatherings, semi-gatherings, rings, fields, modules, vector spaces, algebras, etc. It is the core of the mathematics with many fascinating and expected subjects of examination like gathering theory, ring theory, module theory, and multiplicative ideal theory, etc. The part of mathematics that reviews rings is known as ring theory. Ring scholars study properties of numerical structures, for example, integers and polynomials. The universality of rings makes them a focal arranging rule of contemporary mathematics.

- **Commutative rings:** Commutative rings are vastly improved perceived than non-commutative ones. Algebraic calculation and algebraic number theory, which give numerous characteristic instances of commutative rings, have driven a significant part of the advancement of commutative ring theory, which is currently, under the name of commutative algebra, a significant region of present day mathematics.

  Since these three fields (algebraic calculation, algebraic number theory and commutative algebra) are so personally associated it is normally troublesome and unimportant to choose which field a specific outcome has a place with. For instance, Hilbert's Nullstellensatz is a hypothesis which is key for algebraic math, and is expressed and demonstrated regarding commutative algebra. Additionally, Fermat's last hypothesis is expressed regarding rudimentary number-crunching, which is a piece of commutative algebra, yet its verification includes profound consequences of both algebraic number theory and algebraic math.

- **Noncommutative rings:** Non-commutative rings are very extraordinary in flavor, since more strange conduct can emerge. While the theory has created in its own right, a genuinely late pattern has tried to resemble the commutative improvement by building the theory of specific classes of non-commutative rings in a mathematical manner as though they were rings of capacities on (non-existent) 'non-commutative spaces'. This pattern began during the 1980s with the improvement of non-commutative calculation and with the revelation of quantum gatherings. It has prompted a superior comprehension of non-commutative rings, particularly non-commutative Noetherian rings.

### 1.1 Ring theory

A ring is a set outfitted with two activities (ordinarily alluded to as expansion and augmentation) that fulfill certain properties: there are added substance and multiplicative personalities and added substance inverses, option are commutative, and the tasks are acquainted and distributive. The investigation of rings has its foundations in algebraic number theory, through rings that are speculations and augmentations of the integers, just as algebraic math, by means of rings of polynomials. These sorts of rings can be utilized to take care of an assortment of issues in number theory and algebra; one of the soonest such applications was the utilization of the Gaussian integers by Fermat, to demonstrate his popular two-square hypothesis. There are numerous instances of rings in different regions of mathematics also, including geography and numerical examination.

A ring is a set RR along with two activities (+) and (·) fulfills the accompanying properties (ring maxims):

i. RR is an abelian bunch under expansion. That is, RR is shut under expansion, there is an added substance personality (called 0), each component $a \in R$ has an added substance backwards $-a \in R$, and option is affiliated and commutative.

ii. RR is shut under duplication, and increase is cooperative

$$\forall a, b \in R;\ a.b \in R$$

$$\forall a, b, c \in R;\ a.(b.c)=(a.b).c.$$

iii. Multiplication disperses over expansion:

$$\forall a, b, c \in R\ a.(b.c)=a.b+a.c \text{ and } (b.c).a=b.a+c.a.$$

A ring is generally signified by (R, +,.) and frequently it is composed distinctly as R when the tasks are perceived.

### 1.2 Graph theory

Let G = (V, E) be a graph. A graph G is supposed to be associated, if for each pair of unmistakable vertices a, b $\in$V, there exists at any rate one way in G among a and b. Let G = (V, E) be an associated graph. Recall that for unmistakable vertices a, b of G, the separation among an and b, indicated by dG(a, b) or d(a, b) is characterized as the length of a shortest way among an and b in G. We characterize d(a, a) = 0 and characterize the diameter of G meant by diam(G) as $diam(G) = \sup\{d(a, b) | a, b \in V\}$. A straightforward graph G is supposed to be finished if every pair of particular vertices of G is nearby in G. Let $n \geq 1$. A total graph on n vertices is indicated by $K_n$. A bipartite graph is a graph G whose vertex set can be partitioned into two subsets A and B with the end goal that no edge has the two closures in any one subset. A total bipartite graph is a graph G which might be partitioned into two disjoint nonempty vertex sets An and B with the end goal that two unmistakable vertices are adjoining if and just on the off chance that they are in particular vertex sets.

Let R alone a commutative ring with nonzero personality, I(R) be the arrangement of all ideals of R and A*(R) be the arrangement of all non-zero annihilator ideals of R, where an ideal I is supposed to be an annihilator ideal if there exists a nonzero ideal J with the end goal that IJ = 0. The idea of the zero-divisor graph of a commutative ring R, meant by (R), was introduced by Beck in 1988, which was essentially concerned with coloring of rings. The vertices of (R) are the nonzero zero-divisors of R, and two unmistakable vertices x and y are adjoining if and just if x y = 0. The zero-divisor graph (R) has been reached out to commutative semi-groups and non-commutative rings by Redmond. The idea of the annihilating-ideal graph of a commutative ring R back to, who let all components of A*(R) as vertex set, and two unmistakable vertices I and J are nearby if and just if IJ = 0, which is indicated by AG(R). In ring theory, the structure of a ring R is intently attached to ideal's behavior more than components', thus it deserves to characterize a graph whose vertices are ideals rather than components.

## 2. ANNIHILATING-IDEAL GRAPHS AG(R) OF COMMUTATIVE RINGS

We observe that if $R_1$ and $R_2$ be two limited commutative rings with the end goal that $R_1 \cong R_2$, at that point $AG(R_1) \cong AG(R_2)$. Therefore, it follows that $\dim(AG(R_1)) = \dim(AG(R_2))$.

**Remark 1:** However, the converse isn't true in general. For instance, consider $R_1 = Z_2 \times Z_2$ and $R_2 = Z_3 \times Z_3$. At that point $AG(R_1) = AG(R_2) = P_2$, however $R_1$ isn't isomorphic to $R_2$.

**Theorem 1:** For a commutative ring R

- dim(AG(R)) is finite if and only if dim(Γ(R)) is finite.
- dim(AG(R)) is undefined if and only if R is an integral domain.

**Proof:**

(i) Expect that dim(Γ(R)) is limited. R is a limited ring and therefore dim(AG(R)) is limited.

Conversely, assume dim(AG(R)) is limited, express equivalent to k. At that point the number of vertices in AG(R) is not exactly or equivalent to $3^k + k + 2$ as $\mathrm{diam}(AG(R)) \leq 3$, that is, $|A^*(R)| \leq 3^k + k + 2$. So R is limited and therefore dim(Γ(R)) is limited.

(ii) This follows from the way that an integral area has no non-zero annihilating ideals.

**Theorem 2:** Let R be a ring. At that point dim(AG(R)) is limited if and just if every vertex of AG(R) has limited degree.

**Proof:** Let dim(AG(R)) be limited, state dim(AG(R)) = k. At that point r(v) is a k − tuple each coordinate of which has a place with the set $\mathfrak{D} = \{0, 1, 2, 3\}$ as $\mathrm{diam}(AG(R)) \leq 3$. Consequently, the number of vertices in AG(R) is all things considered $4^k$. Therefore, every vertex has a limited degree. Conversely, assume that every vertex of AG(R) has limited degree. As $\mathrm{diam}(AG(R)) \leq 3$, therefore AG(R) is a limited graph.

## 3. FINITENESS CONDITIONS OF ANNIHILATING-IDEAL GRAPHS

For a ring R, soc(R) is the total of all negligible ideals of R (if there are no insignificant ideals, this whole is characterized to be zero). Additionally, if X is a component or a subset of a ring R, we characterize the annihilator of X in R by $\mathrm{Ann}(X) = \{r \in R \mid rX = (0)\}$. An ideal I of a ring R is called an annihilator ideal on the off chance that I = Ann(x) for some $x \in R$. Additionally, for a set

A we signify the cardinal number of A by |A|. Leave R alone a ring. We state that the annihilating-ideal graph AG(R) has ACC (respectively, DCC) on vertices if R has ACC (respectively, DCC) on A*(R).

**Theorem:** Let R be a non-domain ring. At that point AG(R) has ACC (respectively, DCC) on vertices if and just if R is a Noetherian (respectively, an Artinian) ring.

**Proof:** Suppose that AG(R) has ACC (respectively, DCC) on vertices. Let 0 ≠ x ∈ Z(R) and P = Ann(x). At that point

$$\{I : I \trianglelefteq R,\ I \subseteq Rx\} \cup \{I : I \trianglelefteq R, I \subseteq P\} \subseteq \mathbb{A}(R)$$

It follows that the R-modules Rx and P have ACC (respectively, DCC) on sub-modules for example Rx and P are Noetherian (respectively, Artinian) R-modules. Since Rx ≅ R/P, by, R is a Noetherian (respectively, an Artinian) ring. The converse is clear. Taking into account above theorem and Cohen's theorem, one may naturally ask, when Spec(R) ∩ A(R)* ≠ ∅ and every prime ideal P ∈ A(R)* is limitedly generated, is R Noetherian? The answer is no! The accompanying example gives a non-Noetherian ring R for which every prime vertex of AG(R) is limitedly generated.

- **Example:** Let K be a field and D: = K[{$x_i$: i ∈ Z}]. The domain D is an extraordinary factorization domain and Q = $x_1$D is a principal tallness one prime of D. Let R: = D/$x^2{}_1$D and $P_0$: = Q/$x^2{}_1$D. At that point $P_0$ is a prime ideal of R with $P^2{}_0$ = (0). Subsequently $P_0$ ∈ AG(R) and $P_0$ ⊆ P for every P ∈ Spec(R) Moreover, $x^2{}_1$D is a Q-primary ideal of D. It follows that the zero ideal of R is P0-primary. Consequently IJ = (0) in R with I ≠ (0) and J≠ (0), suggests both I and J are contained in P0 = Q/$x^2{}_1$D. Therefore, $P_0$ = Z(R) is a cyclic ideal and Spec(R) ∩ A(R)* = {$P_0$}, yet R isn't a Noetherian ring.

The accompanying proposition shows that rings R for which every nonzero proper ideal I of R is a vertex of AG(R) are bountiful.

**4. CONNECTIVITY OF THE ANNIHILATING-IDEAL GRAPHS**

By Anderson and Livingston, for every ring R, the zero-divisor graph Γ(R) is an associated graph and diam(Γ(R)) ≤ 3. Moreover, in the event that Γ(R) contains a cycle, at that point gr(Γ(R)) ≤ 4. These realities later were created by Behboodi for modules over a commutative ring, by Redmond, for the undirected zero divisor graphs of a non-commutative ring and by Behboodi and Beyranvand for the strong zero-divisor graphs of non-commutative rings. Here we will show that these realities are additionally true for the annihilating-ideal graph of a ring. Let S be a commutative multiplicative semi group with 0 (0x = 0 for all x ∈ S). The zero-divisor graph of S (meant by Γ(S)) is a graph whose vertices are the nonzero zero-divisors of S, with two unmistakable vertices a, b joined by an edge in case stomach muscle = 0. In, it is indicated that Γ(S) is constantly associated, and diam(Γ(S)) ≤ 3. Next, we utilize this to acquire a similar result for the annihilating ideal graph of a ring.

**Theorem:** For every ring R, the annihilating-ideal graph AG(R) is associated and diam(AG(R)) ≤ 3. Moreover, in the event that AG(R) contains a cycle, at that point gr(AG(R)) ≤ 4.

**Proof:** Let R be a ring. Clearly, the set S: = A(R) is a commutative semi-group under augmentation and furthermore AG(R) = Γ(S). In this way AG(R) is an associated graph and diam(AG(R)) $\leq$ 3. Presently, assume that AG(R) contains a cycle, and let C: = $I_1$ ───….───$I_n$───$I_1$ be a cycle with the least length. On the off chance that $n \leq 4$, we are finished. Otherwise, we have $I_1 \cap I_4 \neq (0)$. We have to just consider three cases:

- **Case 1:** $I_1 \cap I_4 = I_1$. At that point $I_1I_3 \subseteq I_4I_3 = (0)$ and $I_1$ ─── $I_2$ ─── $I_3$ ─── $I_1$ is a cycle, a contradiction. The case $I_1 \cap I_4 = I_4$ is similar.

- **Case 2:** $I_1 \cap I_4 = I_2$. At that point $I_2 \subseteq I_1$, $I_2I_n = (0)$ thus $I_2$ ───….─── $I_n$ ─── $I_2$ is a cycle with length $n - 1$, a contradiction. The case $I_1 \cap I_4 = I_3$ is similar.

- **Case 3:** $I_1 \cap I_4 = I_1, I_2, I_3, I_4$. At that point we have $I_2 (I_1 \cap I_4) = (0)$, $I_3 (I_1 \cap I_4) = (0)$ and $I_2$ ─── $(I_1 \cap I_4)$ ─── $I_3$ ─── $I_2$ is a cycle, a contradiction. In this way $n \leq 4$, for example gr(AG(R)) $\leq 4$.

All rings R for which the graph AG(R) has a vertex neighboring every other vertex. At that point we apply this to characterize rings R for which the graph AG(R) is complete or star.

## 5. R HAS EXACTLY ONE MAXIMAL N-PRIME OF (0)

Throughout we consider rings R with the end goal that R has precisely one maximal N-prime of (0). We prove a few results on $(AG(R))^c$ regarding its connectedness, its girth, determination of its vertices which concede a supplement.

**Lemma:** Let R be a ring with the end goal that R concedes a maximal N-prime P of (0) which isn't a B-prime of (0) in R. Then $(AG(R))^c$ is associated and diam($(AG(R))^c$)=2.

**Proof:** We know that $(AG(R))^c$ is associated and diam($(AG(R))^c$) $\leq 2$. It is notable that AG(R) is associated. It is noted in the proof that A(R)* has at any rate two components. Presently it follows that diam((AG(R))c) $\geq 2$. Subsequently we get that diam($(AG(R))^c$) = 2.

## 6. CONCLUSION

Rings considered in this article are commutative with character which concedes at any rate one non-zero annihilating ideal. For such a ring R, we determine necessary and adequate conditions all together that the supplement of its annihilating ideal graph is associated and furthermore discover its diameter when it is associated. We introduce and examine the annihilating-ideal graph of R, meant by AG(R). It is the (undirected) graph with vertices A(R)*:= A(R)\{(0)}, and two unmistakable vertices I and J are neighboring if and just if IJ = (0). The investigation of rings has its foundations in algebraic number theory, through rings that are speculations and augmentations of the integers, just as algebraic math, by means of rings of polynomials.

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